JOURNAL OF MATERIALS SCIENCE **32** (1997) 6495–6503

Comparative statistical analysis of the fatigue of composites under different modes of loading

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This paper deals with fatigue testing of a 16-ply aerospace laminate based on T800 Toray carbon fibres in Narmco 5245 epoxy resin, under diverse loading regimes including constant-stress bending and tension and block loading in tension–tension and tension–compression. Statistical analysis of the results has been performed by means of the extreme value theory, confirming a Weibull domain of attraction for each sample distribution. This procedure permits the assessment of low probabilities of failure, which can be used as characteristic values for practical design.

1. Introduction

The large scatter shown by experimental measurements of the mechanical properties of composites, especially those related to fatigue, demands the use of statistical methods in order to achieve their analysis and prediction for design. Since engineering design is based on low probabilities of occurrence, which determine the safety of the element, a suitable theory, such as the extreme value theory, has to be applied [1].

Among the three possible statistical extreme distributions to be considered, namely, Frechet, Weibull and Gumbel, the Weibull distribution has been widely used as a feasible model for analysis of the phenomena relating time and failure, a typical problem in materials studies, as the only one fulfilling all of the statistical requirements [2, 3].

Two different kinds of Weibull distribution, for maximum or for minimum values, can be envisaged, the latter being the appropriate one, in relation to the problem we are confronted with, where the lowest numbers of cycles to failure are determinants.

Considering the number of cycles to failure, N, as the random variable, the Weibull probability distribution function for minima is expressed as

$$F(N;\, N_o, N_u, \alpha_f) = 1 - \exp\left[-\left[\frac{N - N_u}{N_o}\right]^{\alpha_f}\right] \quad (1)$$

for $N \geqslant N_u$, $0 < N_u < \infty$, $N_o > 0$, $\alpha_f > 0$, where $F(N; N_o, N_u, \alpha_f)$ represents the probability of failure for each value of N, N_u is the location parameter or initial time in which no failures occur, N_o is the scale parameter or characteristic sample life and α_f is the shape parameter.

Because of the resulting simplicity in the mathematical treatment in addition to its satisfactory application to current problems on composite fatigue, the Weibull distribution function is generally preferred [4–7].

In the special case in which the location parameter is zero, the Weibull distribution reverts to a biparametric function, given by the equation

$$F(N;\, N_o, \alpha_f) = 1 - \exp\left[-\left[\frac{N}{N_o}\right]^{\alpha_f}\right] \quad (2)$$

for $N \geqslant 0$, $N_o > 0$, $\alpha_f > 0$. Nevertheless, Talreja shows in [8] that when a sample belonging to a three-parameter Weibull distribution is represented for simplicity by a two-parameter Weibull distribution, the parameters estimated with this simplification, the shape and scale parameters, adopt greater values than the real ones, the estimation error increasing proportionally with the true value of the location parameter of the sample.

Consequently, in this study the true value of the location parameter has been estimated for each testing mode. This has led to the calculation of the shape and scale parameters of the sample by using the maximum likelihood method, for which the central idea consists of assuming that the sample comes from a population with parent distribution belonging to a parametric family and choosing the parameter values that maximize the probability of occurrence of the sample data.

In this work, the estimations of the parameters and the plots on probability papers have been carried out by means of the expert system EXTREMES of Castillo *et al.* [9].

2. Material and testing procedures

2.1. Experimental material

The material used in the test programme was a modern aerospace laminate made at the Royal Aerospace Establishment, Farnborough (MoD, UK), for the

School of Materials Science of the University of Bath. It was supplied in panels 2 mm thick with a fibre volume fraction, V_f, of 0.69. The manufacturers' mechanical properties for the fibre and the resin are given in Table I.

In the manufacturing process of the laminates a symmetric 16-ply sequence, $[(\pm 45, 0_2)_2]_s$, was constructed from pre-impregnated carbon fibres of intermediate modulus (IM) Toray, T800, in a Narmco 5245 resin described by the manufacturer as an epoxy/bismaleimide.

2.2. Testing procedures

To determine the mechanical characteristics of the laminate, specimens for tensile, compressive, flexural and inter-laminar shear strength tests were cut following ASTM standards. The mechanical test results are summarized in Table II. The fatigue programme carried out on laminate T800/5245 corresponds to tests with blocks of one and four load units in tension, blocks of four units in tension/compression and flexure in four-point loading.

In accordance with [5], in the tension/tension constant-stress tests, five levels of maximum tension were selected, corresponding to 86, 80, 74, 67 and 62% of the tensile strength of the laminate $\sigma_t = 1.63$ GPa.

To simulate real loading conditions on the material, series of dynamic tests of four distinct units of load were programmed, in which, as indicated in Figs 1 and 2, the position of the different units of levels in the block was varied. The length of the units adopted was equal to 5% of the characteristic value of the median life at each stress level.

Finally, the experimentation in four-point loading dynamic flexure was carried out for six levels of maximum tension, corresponding to 65, 70, 75, 80, 85 and 90% of the flexural strength of the laminate, $\sigma_f = 1.08$ GPa [7]. The dimensions of the test set-up can be seen in Fig. 3.

All of the tests were carried out in the laboratory of the School of Materials Science of the University of Bath at frequencies of about 3 and 4 Hz and maintaining an R ratio, $\sigma_{min}/\sigma_{max}$, of 0.1 or 10 for tests in repeated tension or repeated compression.

TABLE I Mechanical properties of laminate components

Material	Tensile modulus (GPa)	Tensile strength (MPa)	Failure strain (%)
Torayca T800 fibre	294	5600	1.9
Narmco 5245 resin	3.3	83	2.9

TABLE II Mechanical strengths of the T800/5245 $[(\pm 45, 0_2)_2]_s$ laminate (error values quoted are standard deviations)

Tensile strength, σ_t	1.63 ± 0.11 GPa
Compressive strength, σ_c	0.88 ± 0.08 GPa
Flexural strength, σ_f	1.08 ± 0.10 GPa
Inter-laminar shear strength, τ_i	60.0 ± 2.80 MPa

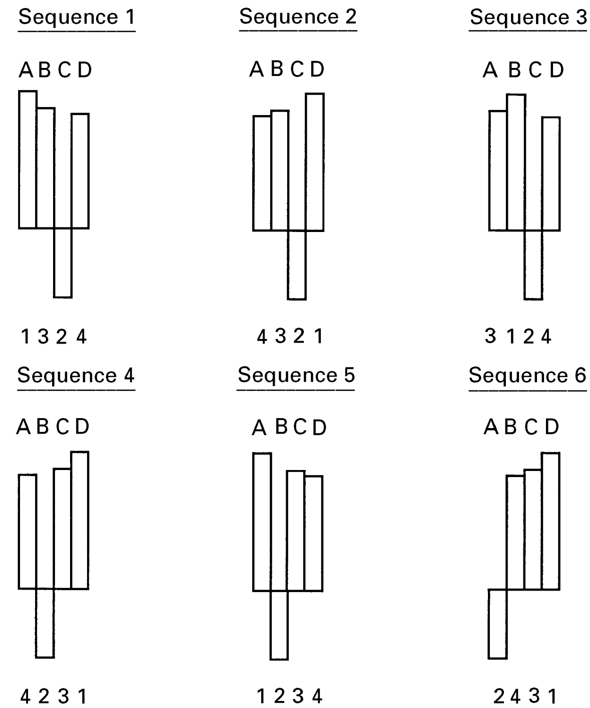

Figure 1 Diagram of load series in the tension/compression test with blocks of four units (1 = 1.3 GPa, 2 = − 0.635 GPa, 3 = 1.1 GPa and 4 = 1.0 GPa).

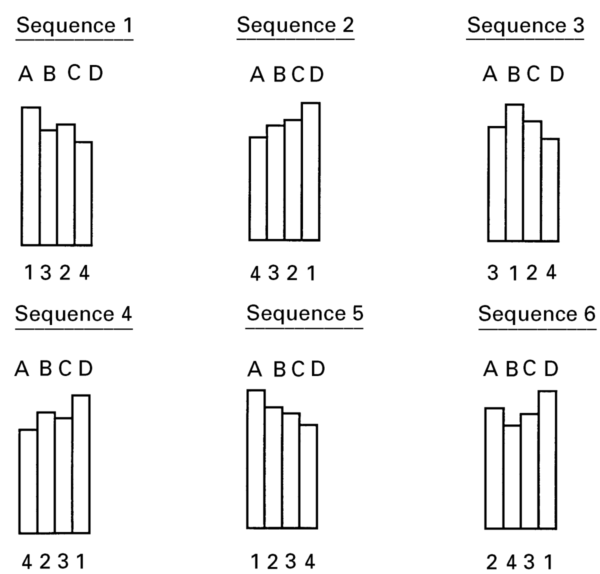

Figure 2 Diagram of load series in the tension/tension test with blocks of four units (1 = 1.3 GPa, 2 = 1.2 GPa, 3 = 1.1 GPa and 4 = 1.0 GPa).

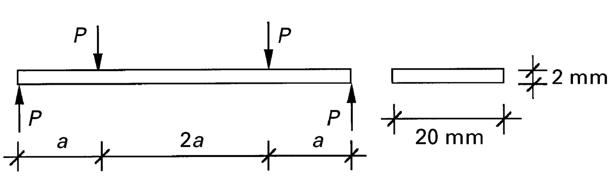

Figure 3 Diagram of flexural tests in four-point loading.

For the flexural tests a servo-hydraulic Dartec M 1000 A machine with a load capacity of ± 5 kN and a maximum displacement of 100 mm, was used, while the rest of the tests were carried out in a servo-hydraulic Instron machine with a load capacity ± 100 kN and ± 50 mm maximum displacement.

3. Normalization of the data samples

As the tests used samples of reduced size, it was thought appropriate to standardize each of the samples (test data sets) with the aim of forming a unique population of variables, assuming that the shape parameter remains constant at all stress levels [10]. This hypothesis was ultimately proved, verifying the parallelism of the fitted Weibull functions at each stress level.

In the cases of blocks of four units in tension and blocks of four units in tension/compression it was necessary, before normalization, to determine the equivalent number of cycles referred to any one of the stress levels of the block loading; in this work this was selected as the stress level of failure, as if the loading had been maintained at this level from the start of the test.

The calculation of the equivalent number of cycles in the present case of multi-step loading with different stress ranges assumes the existence of "isodamage lines", i.e. lines along which the fatigue damage is the same, independent of the stress level [11, 12] as

$$\frac{n_A}{N_A} = \frac{n_B}{N_B} = \cdots = \frac{n_I}{N_I} = \cdots = \frac{n_Z}{N_Z} \tag{3}$$

where $n_A, n_B, \ldots, n_I, \ldots, n_Z$ are the number of cycles to failure at the stress levels A, B, ..., I, ..., Z and N_A, $N_B, \ldots, N_I, \ldots, N_Z$ are the representative values, normally given by the median of the number of cycles to failure at the same stress levels.

This assumption, based on the Palmgren–Miner linear damage rule, enables the multi-step loading to be handled statistically in the same way as a one-step test by working out the equivalent numbers of cycles in the different block tests performed; this can be seen in Table III for the loading sequence indicated, in which the conversion of the number of cycles to a generic level Z follows the expression

$$n_Z(2) = \sum_{I=A}^{Z-1} \left(n_I \frac{N_Z}{N_I} \right) + n_Z(1) \tag{4}$$

$n_Z(1)$ and $n_Z(2)$ being, respectively, the start and the end states of the block loading unit (Fig. 4).

TABLE III Equivalent number of cycles for loading sequence 1 of tension/compression tests with blocks of four units (Fig. 1)

Failure unit	No. of cycles to failure				Equivalent No. cycles
	n_A	n_B	n_C	n_D	
A	429	29 256	4016	162 576	1122
A	533	58 512	8032	325 152	1920
A	394	29 256	4016	162 576	1087
C	462	58 512	4023	162 576	24 097
A	636	58 512	8032	352 152	2023
C	693	87 768	8039	352 152	40 826

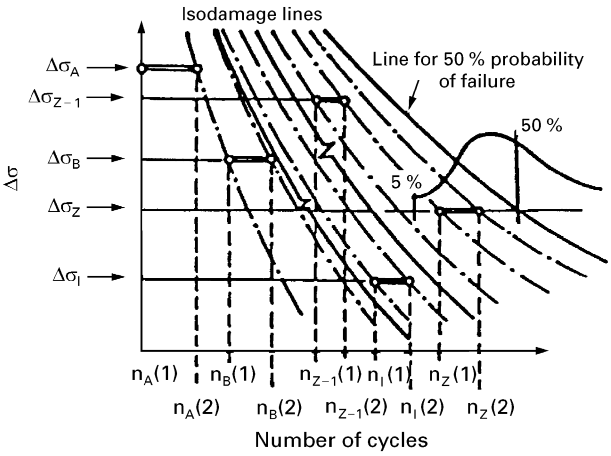

Figure 4 Diagramatic representation of the progress of a multi-step test by means of isodamage lines.

The normalization procedure adopted consisted of the three following stages:

1. Estimation of a two-parameter Weibull function for the samples $N_{ij}(N_{i1}, N_{i2}, \ldots, N_{in})$

$$F(N_i) = 1 - \exp\left[-\left[\frac{N_i}{N_{oi}} \right]^{\alpha_f} \right] \tag{5}$$

where the subscript i represents the load level at which the sample was tested, and the subscript j the order of the variable.

2. Normalization of the samples with respect to the estimated characteristic values, N_{oi}

$$X_{ij} = \frac{N_{ij}}{N_{oi}} \tag{6}$$

3. Estimation of the Weibull distribution function for all of the normalized values X_{ij}.

$$F(X_{ij}) = 1 - \exp\left[-\left[\frac{X_i}{X_o} \right]^{\alpha_f} \right] \tag{7}$$

Although in the initial estimations of shape and scale parameters, a location parameter of zero was assumed, later, after finishing the individual analyses of each kind of test, in the cases where it was necessary this was corrected and the real values of the parameters were determined.

4. Selection of domains of attraction

Determination of the domain of attraction of a parent population is crucial and has a direct influence on safety, particularly when extrapolation is involved. According to Castillo [1], for any continuous distribution function, $F(x)$, only the Frechet, Gumbel and Weibull families are possible as distribution limits for maxima and minima.

Various methods exist which enable us to know the domain of attraction in a distribution function, $F(x)$, when the latter is known. However, the most common problem in practice consists in using a sample of variables without knowing its function $F(x)$. In these cases, as for the one we are dealing with here, other means must be used to identify the kind of domain or the kind of family limit. In this task the probability-paper method and the curvature method will be used.

4.1. The probability-paper method

The basic idea of probability papers is to modify the design scales of the random variable and the probability in such a way that in representing in them a parametric family of distribution functions, this family is converted into a family of straight lines. The first step in identifying the domain of attraction of a distribution function is to represent its values on a Gumbel probability paper for maxima or minima, as appropriate, bearing in mind that in the case of the problem of extremes only the values of the tails can determine the family limit to which the distribution belongs.

Therefore, in order for a minimum extreme value distribution to belong to the Weibull domain of attraction, its representation on Gumbel probability paper for minima must show convexity in its left-hand tail [9]. As is shown in Figs 5, 6, 7 and 8, the standardized samples of each of the fatigue tests carried out on laminate T800/5245 can be interpreted as belonging to a family of Weibull distribution functions.

4.2. The curvature method

This method was developed by Castillo and Galambos and later improved by Castillo *et al.* [13]. The basic idea is to measure the curvature in the tail of interest by means of the ratio of the mean slopes in two neighbouring zones of the tail. To calculate the mean slopes of the zones, the curvature method proposes fitting two straight lines by the least-squares

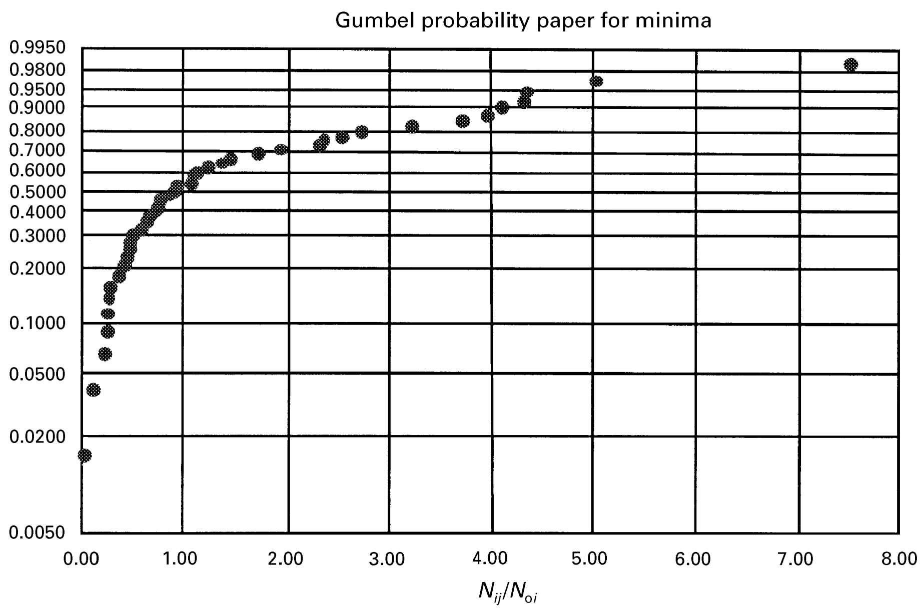

Figure 5 Weibull domain of attraction for the normalized sample of results from the tension/tension tests.

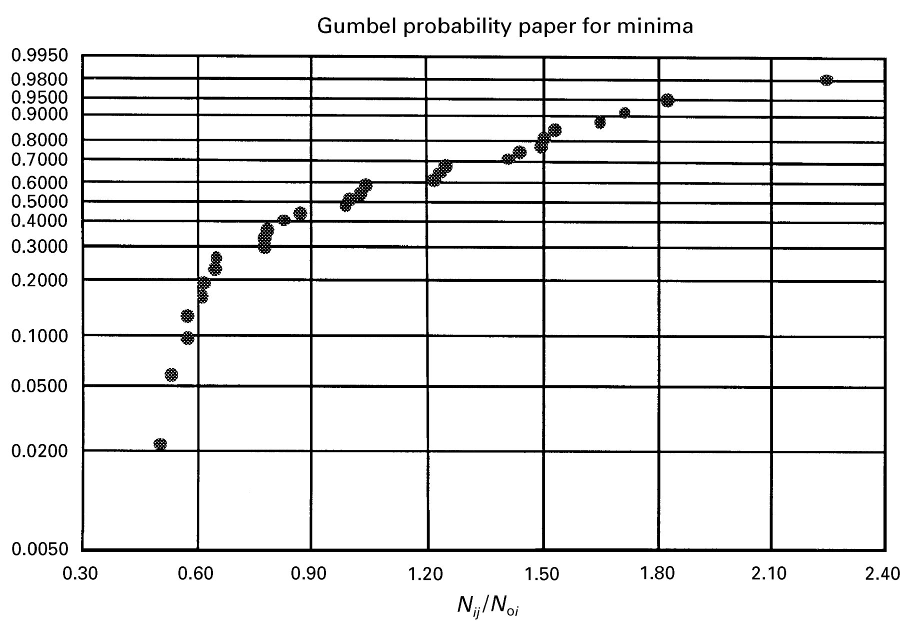

Figure 6 Weibull domain of attraction for the normalized sample of results from the tension/tension tests with load applied by blocks.

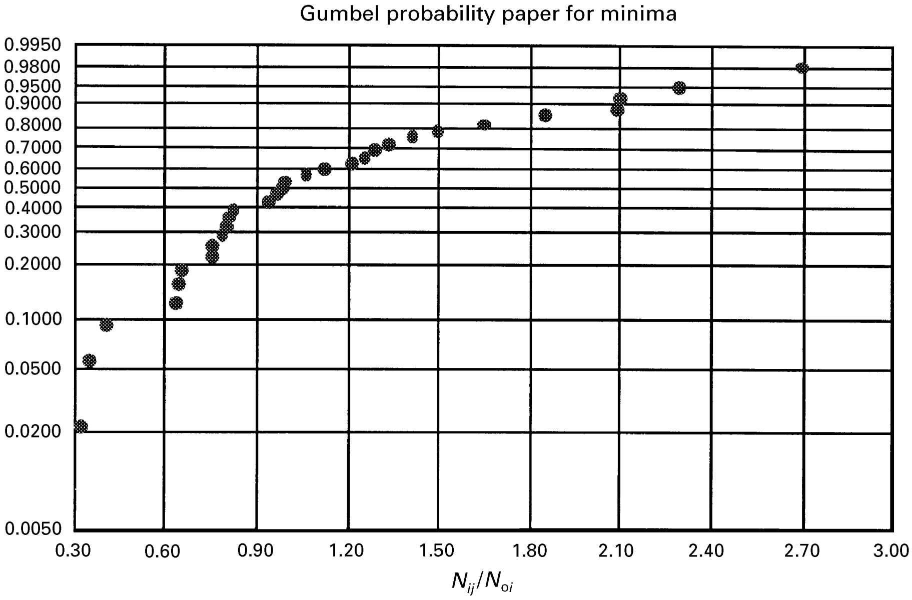

Figure 7 Weibull domain of attraction for the normalized sample of results from the tension/compression tests with load applied by blocks.

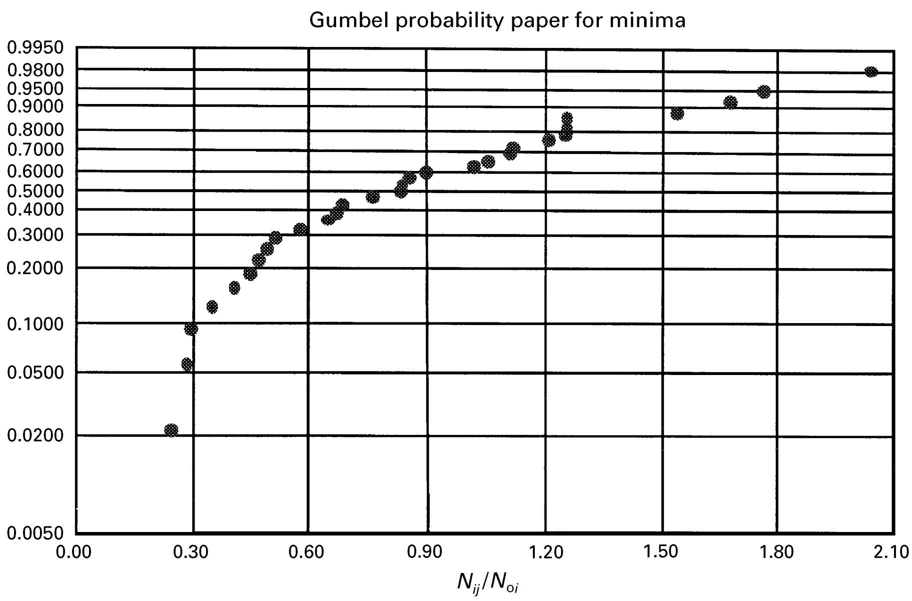

Figure 8 Weibull domain of attraction for the normalized sample of results from the flexural fatigue tests.

method and using, as a measure of the curvature, the ratio, S, of the slopes of these two lines

$$S = \frac{S_{n_1, n_2}}{S_{n_3, n_4}} \qquad (8)$$

where $S_{i,j}$ are the slopes of the straight lines fitted on Gumbel probability paper to order statistic r with $i \leqslant r \leqslant j$. The selection of the values n_1, n_2, n_3 and n_4 is based on the sample size and the speed of convergence.

With this method a Weibull domain of attraction can be determined if S is greater than unity, which has been verified for all of the normalized samples from the different tests.

5. Estimation of parameters

Once the domain of attraction was determined, the samples were represented on Weibull probability paper and the scale and shape parameters were estimated, with the initial supposition of a location parameter equal to zero.

Whenever the Weibull representation was not totally linear, its curvature was corrected to attain the desired linearity, and with the value of the location parameter attained, the scale and shape parameters were re-estimated.

In this way a Weibull distribution function was obtained with three parameters for each of the tests, as shown in Table IV and in Figs 9, 10, 11 and 12.

6. Conclusions

1. The shape parameters of the samples, i.e. the slopes of the straight regression lines fitted on the Weibull paper, will be the comparison parameters of the four types of fatigue tests carries out on laminate T800/5245. If, in all of these, this parameter has a

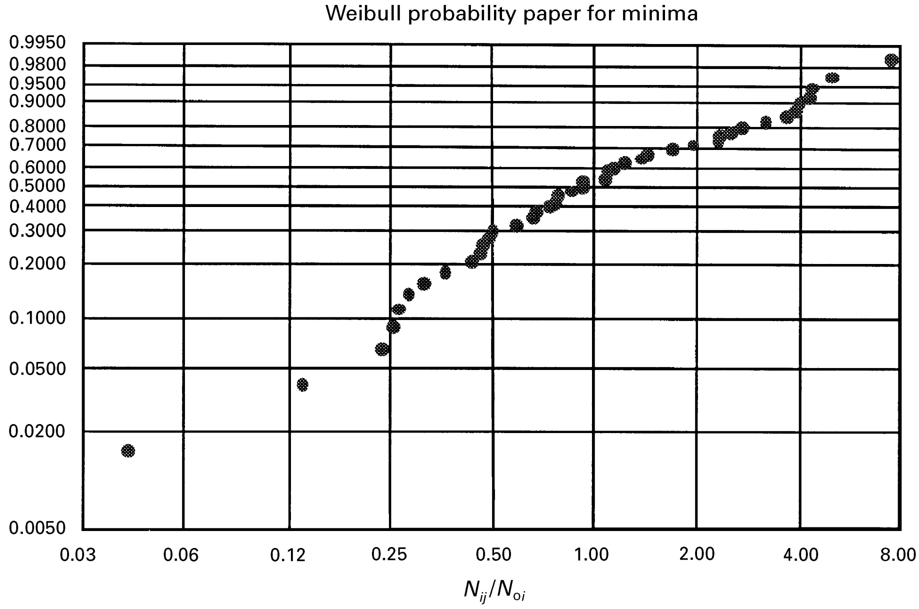

Figure 9 Weibull representation for the normalized sample of results from the tension/tension tests (estimated Weibull parameters: $X_u = 0$, $X_o = 1.004$, $\alpha_f = 1.04$).

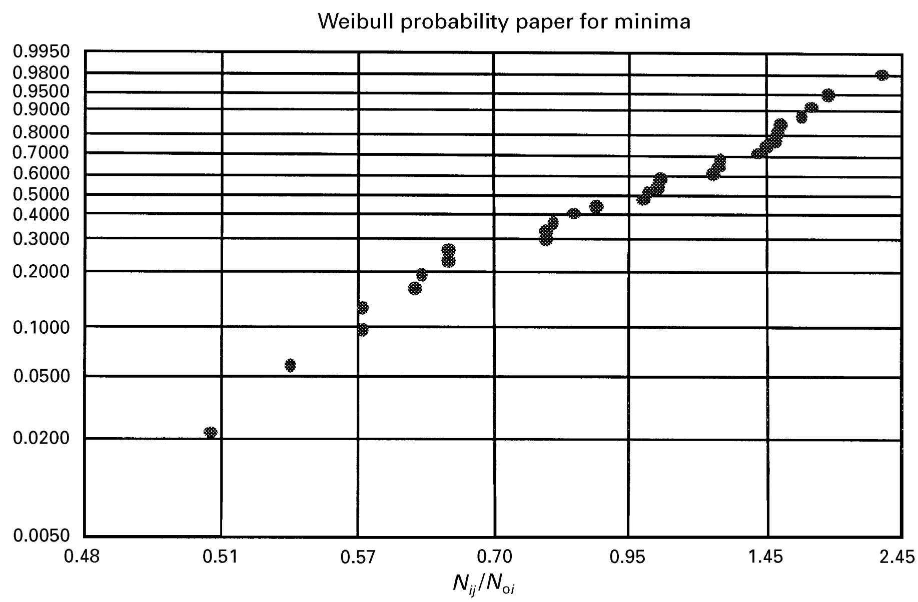

Figure 10 Weibull representation for the normalized sample of results from the tension/tension tests with load applied by blocks (estimated Weibull parameters: $X_u = 0.45$, $X_o = 0.63$, $\alpha_f = 1.32$).

TABLE IV Estimation of the Weibull function of two and three for the normalized results of dynamic tests on laminate T800/5245[$\pm$ 45, $0_2)_2]_s$

Kind of test	Location parameter X_u	Scale parameter X_o	Shape parameter α_f
Tension/tension (1 unit)	0	1.004	1.04
Tension/tension (4 units)	0.45	0.63	1.32
Tension/compression (4 units)	0.25	1.01	1.59
Flexure	0.2	0.76	1.48

similar value, it could be postulated that the damage mechanism of the material coincides, even when the material is submitted to different kind of stress.

From the values of the estimated shape parameters in Table IV, it can be seen that in all cases these are greater than one and less than two. This result suggests a similar damage mechanism in the four kinds of test, a conclusion which can be corroborated by micrographic studies carried out on test samples of different materials, including T800/5245, tested in flexure or with combined tension/compression stresses, where the damage caused by tension always predominates over that caused by compression.

Fig. 13 shows, for a certain number of cycles and load applied, the greater number of cracks in tension than in compression found in the unidirectional layers of a specimen of the T800/5245 laminate, having been subjected to a four-point bending test [7].

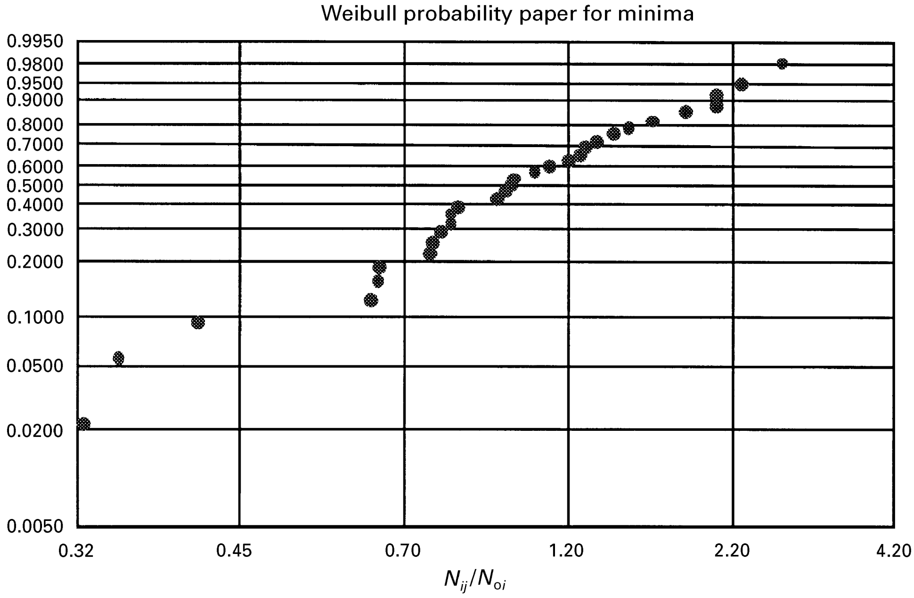

Figure 11 Weibull representation for the normalized sample of results from the tension/compression tests with load applied by blocks (estimated Weibull parameters: $X_u = 0.20$, $X_o = 1.01$, $\alpha_f = 1.59$).

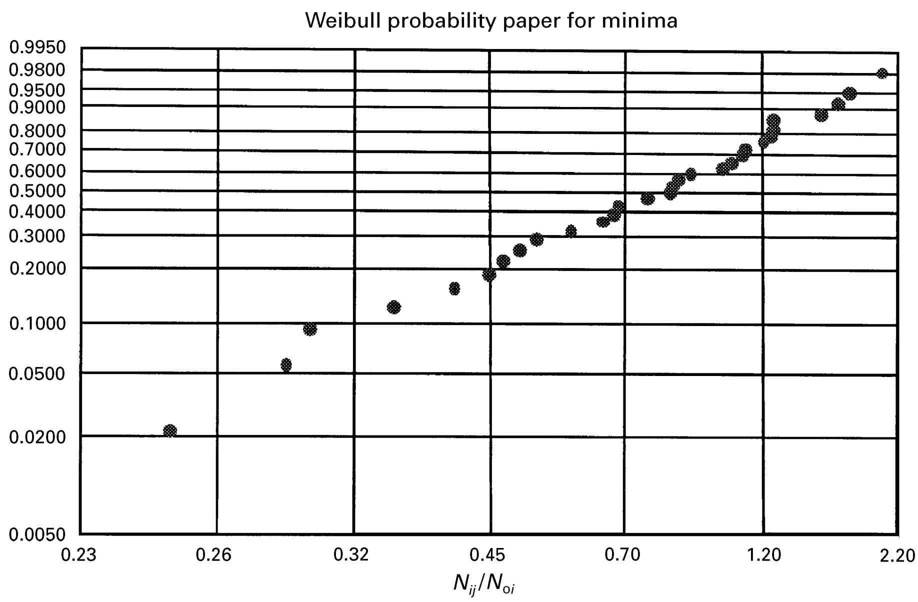

Figure 12 Weibull representation for the normalized sample of results from the flexure fatigue tests (estimated Weibull parameters: $X_u = 0.2$, $X_o = 0.76$, $\alpha_f = 1.48$).

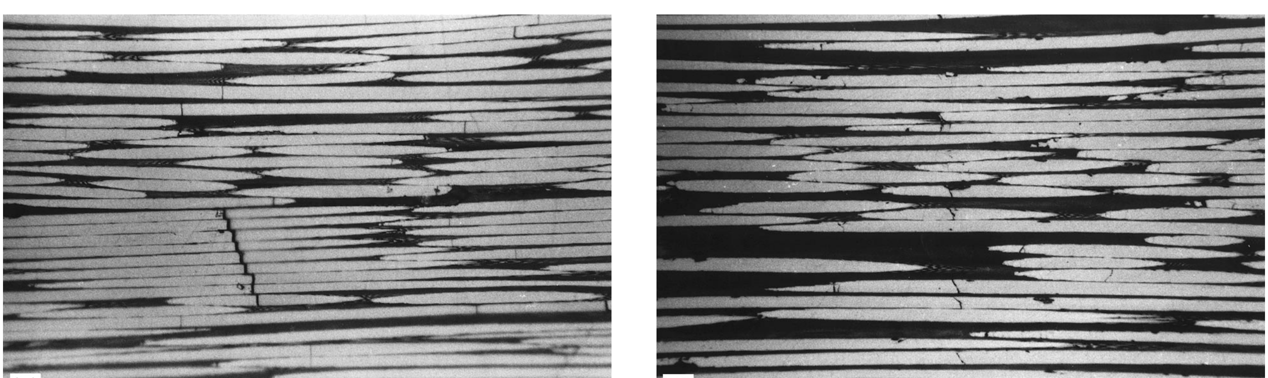

Figure 13 Photographs of the state of damage in tension (a) and compression (b) for the 0° external layers of the T800/5245 $[(\pm 45, 0_2)_2]_s$ laminate in a four-point bending test ($\times 400$).

2. With regard to the calculation of design values, once the distribution function which governs the behaviour of the samples of experimental results is determined, it is easy to obtain the number of load cycles associated with probabilities of failure for the material.

By way of example, the values of numbers of cycles for probabilities of 5% are shown in Tables V and VI, corresponding to the typical design value, and of 50%, or mean lifetime, at six stress levels tested in bending and in pure tension (unit tension/tension).

3. Figs 14 and 15 show the S/Log N curves, determined from the values in Tables V and VI respectively. As can be observed, for maximum stress above 75% of the failure stress, the fatigue life of specimens under pure tension loading turns out to be shorter than that in bending for the same induced stress in the outer ply of the laminate. Below this value, an inversion in the trend of the reciprocal fatigue lives seems to occur.

On the other hand, the fatigue life range measured between the S/Log N curves for probabilities of failure $P = 0.50$ and $P = 0.05$ is broader for the tension tests than for bending. This reveals a more rapid progress in the damage mechanisms in bending, probably imputable to the negative influence of a side of specimen being under compression.

4. The fatigue results obtained from the bending tests permit us to analyse the simultaneous development of damage mechanisms in tension and compression, as an alternative to the pure compression test, in which the failure occurs in a catastrophic unexpected mode. Nevertheless, further research is needed since they cannot be used definitely to draw conclusions on either the fatigue behaviour under pure tension or compression loading, or the interaction of damage caused by combined tension and compression units of load.

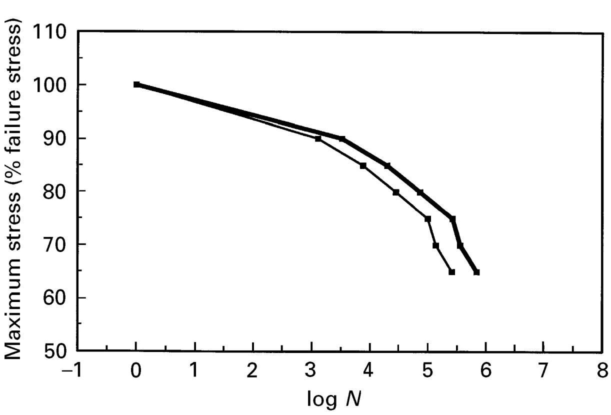

Figure 14 Curve S/Log N in four-point bending test on laminate T800/5245. (—) 50% probability; (—) 5% probability.

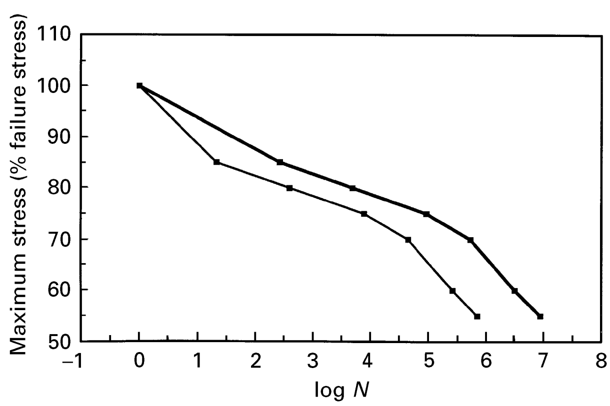

Figure 15 Curve S/Log N in pure tension test on laminate T800/5245. (—) 50% probability; (—) 5% probability.

TABLE V Value of number of cycles for different probabilities of failure in the four-point bending test on laminate T800/5245[($\pm$ 45, 0_2)$_2$]$_s$

Load level (% σ_f)	5% probability of failure (cycles)	50% probability of failure (cycles)
65	261 917	687 992
70	137 867	362 356
75	102 034	268 173
80	28 420	74 706
85	7671	20 159
90	1263	3319

TABLE VI Value of number of cycles for different probabilities of failure in pure tension test on laminate T800/5245[($\pm$ 45, 0_2)$_2$]$_s$

Load level (% σ_f)	5% probability of failure (cycles)	50% probability of failure (cycles)
55	710 461	8 685 890
60	261 420	3 196 040
70	44 522	544 317
75	7674	93 820
80	398	4876
85	21	261

Acknowledgements

The authors gratefully acknowledge the financial aid given by the FICYT and the PRI of Asturias (Spain) and by the Defence Research Agency, Farnborough (UK), as well as permission to use the program EXTREMES and the statistical advice provided by Professor E. Castillo of the University of Cantabria (Spain).

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Received 23 July 1996
and accepted 17 June 1997